REMAINING STRENGTH EVALUATION METHOD OF PLATE GIRDERS WITH CORRODED FLANGE UNDER SLEEPERS

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For steel railway bridges with open deck system, the local buckling of compressive flanges under sleepers on plate girders caused by the local corrosion of flanges leads to the collapse of bridges. The local corrosion is thought to have a substantial impact on the ultimate strength of main girders. However, past research on ultimate strength contains few inquiries into the relationship between the local corrosion of compressive flanges and the ultimate strength of main girders, and a method of evaluating the relationship has not yet been established. This study proposed a remaining strength evaluation method that takes into account the local corrosion under sleepers based on parameter analyses that focus on the strength interaction of plate girders subjected to bending and local loads simultaneously. In addition, the study verified the applicability of this method from existing experimental values and analytical values obtained through FEM to clarify that ultimate strength can be evaluated with a high degree of accuracy.

Key Words : *plate girder, sleeper, local corrosion, remaining strength, local buckling*

1. INTRODUCTION

In Japan, many steel railway bridges with open deck system have been in use for over 60 years, and deterioration and damage due to corrosion is becoming obvious. Partial repairs and reinforcement as well as total replacement of girders have taken place in recent years, but it cannot be said with certainty that these measures have been selected after proper, quantitative evaluations of the remaining strength of the girders. In light of recent socioeconomic conditions, there is an even more pressing need to establish streamlined, economically viable methods for maintaining these bridges.

One phenomenon that requires maintenance is local corrosion of the upper flanges under sleepers, a type of corrosion characteristic of steel railway bridges that it shall be referred to as "corrosion under sleepers" throughout this report. **Fig.1** shows that the coating on upper flanges under sleepers wears more easily and is more susceptible to dampness than other parts due to the train load acting almost directly through the rails and sleepers onto the girders. Local corrosion progresses as a result. Many studies on ultimate strength of plate girders (Basler & Thurlimann[1)], Moriwaki et al.[2),3)], Fujino[4)], Komatsu et al.[5)], etc.) had been conducted by experiments and analyses. However, none of them were taken account of local corrosion despite the fact of taking into account corrosion under the sleepers is vital toward accurately evaluating the remaining strength of this type

Fig.1 Steel railway bridges with open deck system.

of steel railway bridges. In this study, firstly load bearing capacity in terms of both pure bending and local loads is independently evaluated, then based on these results, a proposed strength interaction curve subjected to the two loads simultaneously evaluates the remaining strength of plate girders under conditions of the load combination.

According to Basler's approach[1], the bending collapse mode for the plate girders is determined by one of the three types that compressive flanges buckle as shown in **Fig.2**: vertical buckling, lateral buckling (lateral torsional buckling), and torsional buckling. Of these, it is clear that vertical buckling of flanges occurs when the width-thickness ratio of the web is extremely small, but it does not occur under normal conditions.

This study evaluates bending load bearing capacity focused on compressive flanges, with lateral buckling and torsional buckling as the two types of flange buckling. Here, the basic buckling length of girders with corrosion under sleepers shall be the spacing between lateral bracings (cross beams) for lateral buckling and the sleeper width for torsional buckling (**Fig.3**).

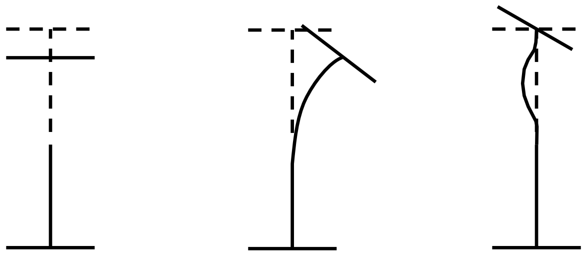


Fig.2 Buckling modes of plate girders.

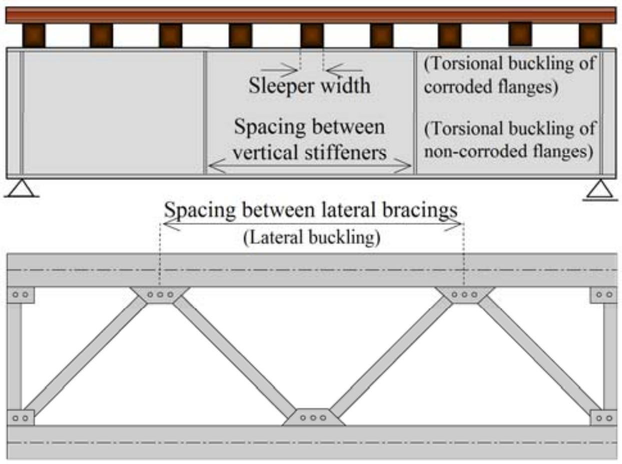


Fig.3 Basic buckling length of girders with corrosion under sleepers.

2. EVALUATION OF BENDING LOAD BEARING CAPACITY

Past studies[6),7)] show that local buckling of compressive flanges due to corrosion under sleepers has a substantial impact on the residual bending load bearing capacity of girders. Also, as previously explained, plate girders with corrosion under sleepers collapse due to the lateral buckling or torsional buckling of compressive flanges.

In light of this information, this study proposes a method of evaluating bending load bearing capacity based on FEM analysis with variable remaining thickness of compressive flanges due to corrosion under sleepers using two types of analytical models that differ for flange length, width and for the number of corroded locations.

This FEM analysis have been performed with ABAQUS, a general-purpose FEM analysis program, and focused only on compressive flanges of plate girders modeled as perfect elasto-plastic material using iso-parametric shell element with 4 nodal points. Material properties are given a yield stress σ_y of 235 MPa, an elastic modulus E of 200 GPa and a Poisson ratio ν of 0.3.

(1) Lateral buckling of compressive flanges

a) Overview of analysis

Fig.4 shows an analytical model. The spacing between lateral bracings (cross beams) is given the basic buckling length L_h in the evaluation of bending load bearing capacity for lateral buckling. It also shows a flange model with corrosion under sleepers at three locations (the shaded areas) within basic buckling length L_h. Both ends are subjected to a uniformly distributed axial compressive stress. Under these conditions, for the flange plate, the web bond parts at both ends have pinned supports, and the bond line between the web and the flange has a simple support to restrain out-of-plane deformation.

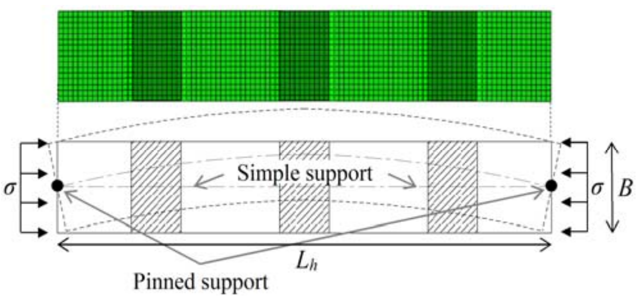


Fig.4 Analytical model (Lateral buckling).

Table 1 shows the analytical parameters: flange width B is 320 mm, flange length (basic buckling length) L_h is 2000 mm in Case 1 and 2600 mm in Case 2, and corrosion is present in three locations in Case 1 and four locations in Case 2. The original thickness of each flange t_0 is 30 mm, and the thickness at the corroded parts under the sleepers is reduced in increments of 3 mm to examine the bending load bearing capacity. These analytical models also take into account residual stress due to initial imperfections and welding.

Table 1 Analytical parameters (Lateral buckling).

		Case 1	Case 2
Flange length	L_h (mm)	2000	2600
Flange width	B (mm)	320	320
Original thickness	t_0 (mm)	30	30
No. of corroded locations	locations	3	4

b) Results of analysis (Lateral buckling)

Table 2 shows the maximum load P_u and maximum axial compressive stress σ_u obtained from this analysis. The entries in the table marked with an asterisk (*) depict cases where torsional buckling preceded lateral buckling in the collapse. The table shows that maximum axial compressive stress σ_u for collapse under lateral buckling is nearly the same as yield stress σ_y. **Fig.5** shows an example of this phenomenon, where the dominant deformation perpendicular to the bridge axis is in a state of ultimate deformation due to the lateral buckling. **Fig.6** shows the load bearing capacity curve of flange lateral buckling: the vertical axis is maximum axial compressive stress σ_u made dimensionless by yield stress σ_y, and the horizontal axis is slenderness ratio parameter λ. The dashed line represents Basler's proposed Equation (1), and Equation (2) expresses slenderness ratio parameter λ. The outlined plots in the figure are well accorded with Basler's load bearing capacity curve under flange lateral buckling (the dashed line). This shows that bending load bearing capacity under flange lateral buckling can be evaluated by deriving bending load bearing stress σ_{uh} from Basler's proposed equation. It is to be noted that the radius of gyration r expressed in Equation (3) is applied to the flange cross-section which takes into account the corrosion wastage in Equation (4).

$$\frac{\sigma_{uh}}{\sigma_y} = 1 - \frac{\lambda^2}{4} \qquad \lambda \le \sqrt{2} \qquad (1)$$
$$= \frac{1}{\lambda^2} \qquad \sqrt{2} < \lambda$$

$$\lambda = \frac{1}{\pi}\frac{L_h}{r}\sqrt{\frac{\sigma_y}{E}} \qquad (2)$$

Here,
σ_{uh} = Bending load bearing stress under lateral buckling
σ_y = Yield stress
λ = Slenderness ratio parameter
L_h = Basic buckling length
r = Radius of gyration
E = Elastic modulus

In addition, radius of gyration r is obtained from Equations (3) and (4).

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{B^2}{12}} \qquad (3)$$

$$I = \frac{B^3 t_f}{12} \qquad (4)$$

Here,
B = Total upper flange width
t_f = Upper flange thickness for corrosion wastage
A = Upper flange cross-sectional area for corrosion wastage

Table 2 Results of analysis (Lateral buckling).

	Case 1		Case 2	
t_f (mm)	P_u (kN)	σ_u (N/mm^2)	P_u (kN)	σ_u (N/mm^2)
30	2120	221	2063	215
27	2003	232	1944	225
24	1808	235	1758	229
21	1590	237	1554	231
18	1372	238	1334	232
15	1144	238	1111	232
12	913	238	888	231
9	*668	*232	664	230
6	*331	*173	*360	*188
3	*121	*126	*157	*164

t_f = Upper flange thickness for corrosion wastage
P_u = Maximum axial compressive force
σ_u = Maximum axial compressive stress

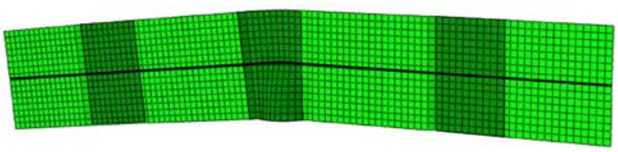


Fig.5 Ultimate deformation state (Lateral buckling).

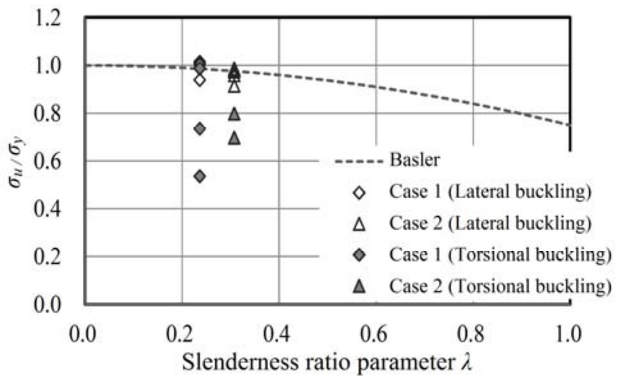


Fig.6 Load bearing capacity curve (Lateral buckling).

(2) Torsional buckling of compressive flanges

a) Overview of analysis

Where there is no corrosion under sleepers, the torsional buckling length is thought to be the spacing between the vertical stiffeners, but when there is corrosion under sleepers, basic buckling length L_t is thought to be the sleeper width of 200 mm. Therefore, the analysis of torsional buckling is focused on corroded compressive flanges under sleepers, and they are modeled focusing on protruding parts of the flanges. **Fig.7** shows the analytical model, where the upper flange plates under sleepers are subjected to forced displacement δ at both ends, and compressive plates with simple support for three sides and free for one side can be assumed. Table 3 shows the analytical parameters: basic buckling length L_t is set as sleeper width, flange half-width b varies in four different cases. In the same way as the analysis described above **2.(1)**, the thickness at the corroded parts of upper flanges under the sleepers is gradually reduced from the original thickness of the upper flange t_0 to examine the load bearing capacity. These analytical models also take into account residual stress due to initial imperfections and welding.

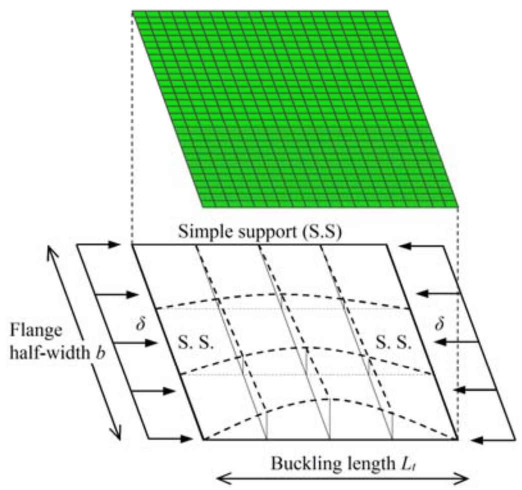


Fig.7 Analytical model (Torsional buckling).

Table 3 Analytical parameters (Torsional buckling).

		Case 1	Case 2	Case 3	Case 4
Sleeper width	L_t (mm)	200	200	200	200
Flange half-width	b (mm)	160	200	300	400
Original thickness	t_0 (mm)	28	30	30	30

b) Results of analysis (Torsional buckling)

Table 4 shows the maximum load P_u and maximum axial compressive stress σ_u obtained from this analysis. The entries in the table marked with an asterisk (*) depict cases where elastoplastic buckling preceded total yield in the collapse. Here, **Fig.8** shows the state of ultimate deformation due to torsional buckling. In addition, **Fig.9** shows the load bearing capacity curve under torsional buckling obtained from the analysis, and Basler's proposed equation. As the figure shows, the analytical values are smaller than those of Basler's proposed equation. This is because buckling coefficient k is set to 0.43 when aspect ratio α is adopted as infinite in Basler's proposed equation. In other words, Basler's proposed equation cannot be applicable in these cases because sleeper width is defined as basic buckling length L_t. Thus, this study proposes Equation (5), which takes account of aspect ratio α based on the analytical results under torsional buckling. The width-thickness ratio parameter R for Equation (5) is expressed in Equation (6).

Note that the fact that basic buckling width L_t is analyzed in this study as sleeper width means that it is limited to situations when corrosion under sleepers between vertical stiffeners is in a single location. **Fig.10 (a)** through **(c)** demonstrates the way of thinking about buckling length based on different corrosion conditions. When corrosion under sleepers between vertical stiffeners is in two locations as in **Fig.10 (c)**, the buckling length shall be considered to be double (for this case, 400 mm).

$$\frac{\sigma_{ut}}{\sigma_y} = 1 \qquad R \le 0.433$$
$$= \left(\frac{0.433}{R}\right)^{0.89} \qquad 0.433 < R \tag{5}$$

$$R = \frac{1}{\pi}\frac{b}{t_f}\sqrt{\frac{12\left(1-\upsilon^2\right)}{k}}\sqrt{\frac{\sigma_y}{E}} \tag{6}$$

Here,

σ_{ut} = Bending load bearing stress under torsional buckling

R = Width-thickness ratio
b = Flange half-width
t_f = Upper flange thickness for corrosion wastage
ν = Poisson ratio
k = Buckling coefficient: $k = 0.43 + (1 / \alpha)^2$
α = Aspect ratio (for this analysis, L_t / b)
σ_y = Yield stress
E = Elastic modulus

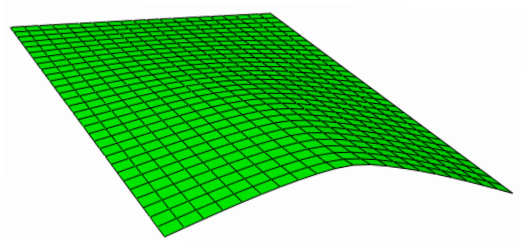


Fig.8 Ultimate deformation state (Torsional buckling).

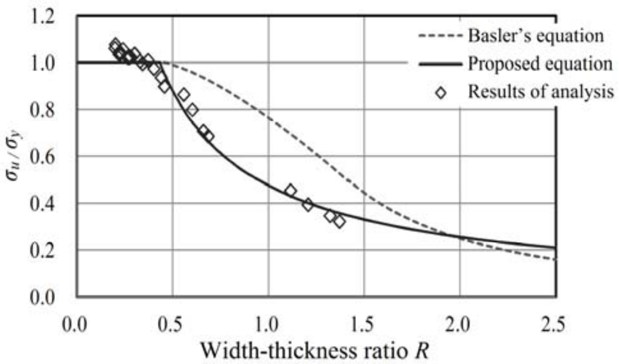
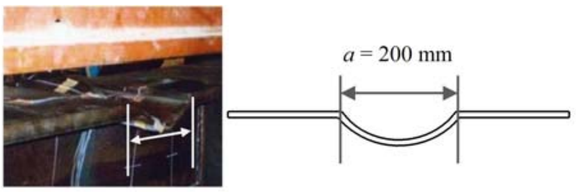
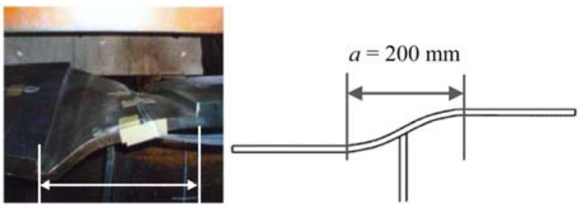


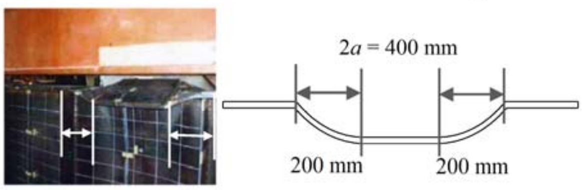
Fig.9 Load bearing capacity curve (Torsional buckling).



(a) When corrosion between vertical stiffeners is in a single location



(b) When corrosion above a vertical stiffener is in a single location



(c) When corrosion between vertical stiffeners is in two locations

Fig.10 Way of thinking about buckling length a.

Table 4 Results of analysis (Torsional buckling).

	t_f (mm)	P_u (kN)	σ_u (N/mm^2)
Case 1	28	1117	249
	25	982	245
	20	771	241
	15	570	237
	10	*325	*203
	5	*85	*107
Case 2	30	1518	253
	25	1240	248
	20	974	243
	15	*687	*229
	10	*376	*188
	5	*93	*93
Case 3	30	2189	243
	25	1801	240
	20	1419	236
	15	*991	*220
	10	*499	*166
	5	*122	*81
Case 4	30	2925	244
	25	2399	240
	20	*1871	*234
	15	*1267	*211
	10	*644	*161
	5	*151	*76

t_f = Upper flange thickness for corrosion wastage
P_u = Maximum axial compressive force
σ_u = Maximum axial compressive stress

3. EVALUATION OF LOCAL LOAD BEARING CAPACITY

Much research has been conducted on calculation equations for the load bearing capacity of girders under local loads. This study showed that Takimoto's proposed calculation equation[8),9)] allowed for accurate evaluations of the load bearing capacity of girders with corrosion under sleepers when they are under local loads[10)]. This study used Equation (7) below, which is further simplified based on Takimoto's equation.

$$P_{u0} = \left(25t_w^{\,2}\sigma_w + 4t_w t_f \sigma_f\right)\cdot\left(1 + \frac{a + 2t_f}{2h_w}\right) \quad (7)$$

Here,
t_w = Web plate thickness
σ_w = Web yield stress
t_f = Upper flange thickness
σ_f = Upper flange yield stress

a = Width on which local load acts
h_w = Web height

4. EVALUATING LOAD BEARING CAPACITY OF GIRDERS UNDER COMBINED LOADS

(1) Overview of analysis

Thus far separate evaluation equations were proposed for load bearing capacity under bending and local loads respectively. Here, it is examined analytically that the ultimate states and load bearing capacity of girders under bending and local loads simultaneously, as is the case with actual bridges. This analysis is conducted by the use of the analytical parameters shown in **Fig.11** and **Table 5**, and by considering combined bending and local loads acting on girders with corrosion in a single central location under sleepers. In addition, **Fig.12** shows the analytical model and boundary conditions at nodal points and sides, and **Fig.13** shows the load conditions that it is applied as a line load to the joint part between the web and the upper flange where there is corrosion under sleepers as well as vertical shear forces to the web cross-sections above the supporting points to balance out the line load. And as shown in **Fig.13**, rigid elements are applied to the end plates of the girder and acted on by couple of forces to work as the bending moment.

These analytical models also take into account residual stress due to initial imperfections in the out-of-plane direction on the web between the vertical stiffeners, and welding between the web and the upper flange.

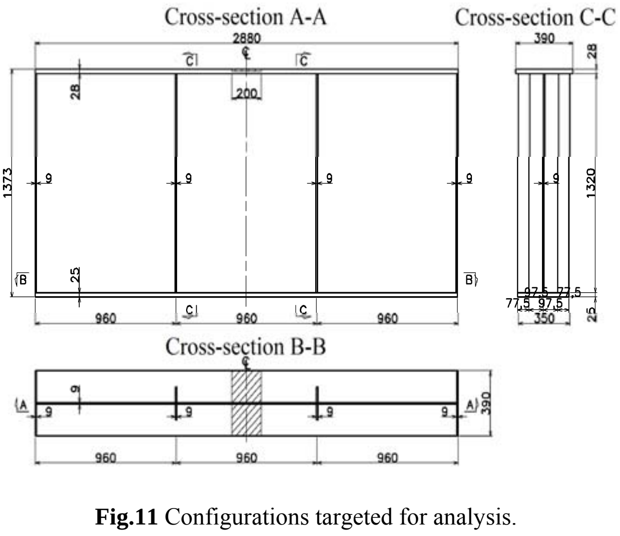
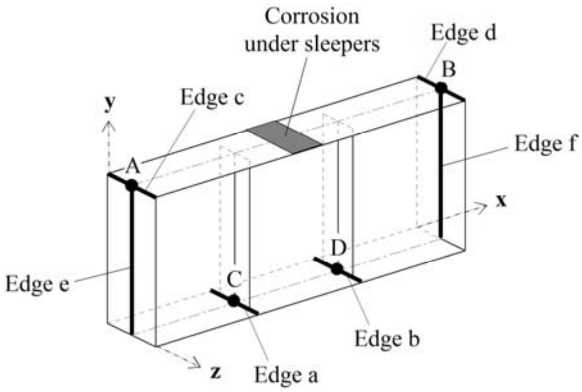


Fig.11 Configurations targeted for analysis.

Table 5 Analytical parameters.

		Case 1	Case 2	Case 3	Case 4
Girder length	L (mm)	2880			
Web height	h_w (mm)	1320			
Web thickness	t_w (mm)	9			
Upper flange width	b_{uf} (mm)	390			
Upper flange thickness	t_{uf} (mm)	28			
Lower flange width	b_{lf} (mm)	350			
Lower flange thickness	t_{lf} (mm)	25			
Width of vertical stiffeners	b_s (mm)	97.5			
Thickness of vertical stiffeners	t_s (mm)	9			
Thickness of corroded portion	t'_{uf} (mm)	28.0*	19.6*	14.0*	8.4*

* Case 1: Non-corroded
* Case 2 to 4: 30, 50, 70% reduction in upper flange original thickness



	x	y	z	Φ_x	Φ_y	Φ_z
A	0	0	1	1	0	0
B	0	0	1	1	0	0
C	1	1	1	1	0	0
D	0	1	1	1	0	0
Edge a	0	1	0	1	0	0
Edge b	0	1	0	1	0	0
Edge c	0	0	0	0	1	0
Edge d	0	0	0	0	1	0
Edge e	0	0	1	1	0	0
Edge f	0	0	1	1	0	0

1: Fixed; 0: Free

Fig.12 Boundary conditions.

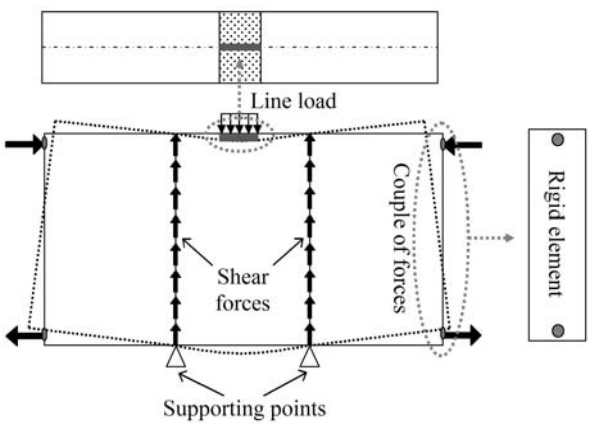


Fig.13 Load conditions.

(2) Results of analysis

Fig.14 and **Fig.15** show the states of ultimate deformation and the von Mises stress at peak load in Case 3 for each of combined and pure bending loads. Most analyses resulted in collapse due to local buckling of the compressive flange as shown in **Fig.14**, but when bending is the superior force or when there is a major reduction in thickness due to corrosion under sleepers, the collapse occurs due to torsional buckling of the flange as shown in **Fig.15**. The stressed condition in **Fig.14** demonstrates that compressive stress on the upper flange occurs and a local load increases marked local stress on the web. In addition, **Fig.15** shows that pure bending concentrates compressive stress on the corroded parts under sleepers.

Fig.16 shows the results of the analysis plotted on an interactive curve. All the analytical values for local load and bending moment on this figure are nearly on the curve that represents Equation (8). Therefore, Equation (8) can be used to evaluate the load bearing capacity of girders under combined loads.

$$\left(\frac{P_u}{P_{u0}}\right)^2 + \left(\frac{M_u}{M_{u0}}\right)^2 = 1 \qquad (8)$$

Here,
P_u = Local load (for combined load)
M_u = Bending moment (for combined load)
P_{u0} = Local load (for individual load)
M_{u0} = Bending moment (for individual load)

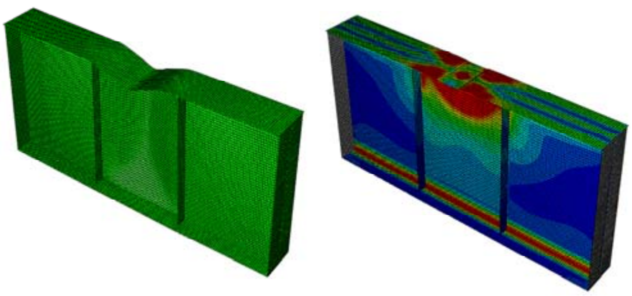


Fig.14 Case 3 – 50% reduction in THK (For combined load).

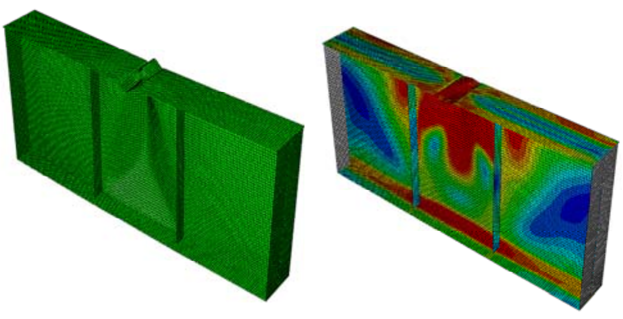


Fig.15 Case 3 – 50% reduction in THK (For pure bending).

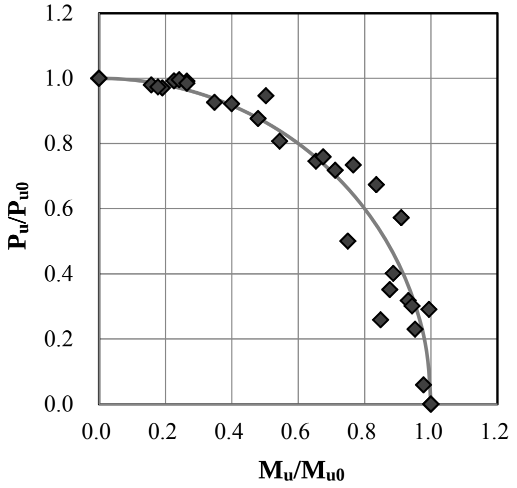


Fig.16 Interactive curve.

5. EVALUATION METHOD OF REMAINING STRENGTH

Thus far this study has examined and evaluated the load bearing capacity of plate girders with corrosion under sleepers under bending (lateral/torsional buckling) and local loads, and the interactive relation of load bearing capacity under combined loads of both forces. Here, this study proposes a specific method for evaluating the remaining strength of plate girders with corrosion under sleepers based on the results of these evaluations. Below is the procedure for this proposal.

First, Equations (1) and (5) are used to calculate the bending load bearing stress for each of lateral and torsional buckling of girders under pure bending force.

$$\frac{\sigma_{uh}}{\sigma_y} = 1 - \frac{\lambda^2}{4} \qquad \lambda \le \sqrt{2}$$
$$= \frac{1}{\lambda^2} \qquad \sqrt{2} < \lambda \qquad (1)$$

$$\frac{\sigma_{ut}}{\sigma_y} = 1 \qquad R \le 0.433$$
$$= \left(\frac{0.433}{R}\right)^{0.89} \qquad 0.433 < R \qquad (5)$$

The smaller value of calculated bending load bearing stresses σ_{uh} or σ_{ut} contributes to the actual state of buckling and determines the applicable bending load bearing stress σ_u for girders. This stress σ_u shall be used in beam theory Equation (9) to calculate the load bearing bending moment M_{u0} under pure bending force.

$$M_{u0} = \frac{\sigma_u}{h} I \quad (9)$$

Here,
I = Second moment of area of corroded part
h = Distance from neutral axis to the top of upper flange of corroded part

Next, the load bearing capacity P_{u0} of girders only under local load can be calculated by using Equation (7).

$$P_{u0} = \left(25t_w^{\,2}\sigma_w + 4t_w t_f \sigma_f\right) \cdot \left(1 + \frac{a + 2t_f}{2h_w}\right) \quad (7)$$

Finally, by using the values of M_{u0} and P_{u0} that are calculated above, and the values of bending moment M_u or local load P_u which acts on actual girders, Evaluation Equation (8) can estimate remaining load bearing capacity of girders under combined loads.

$$\left(\frac{P_u}{P_{u0}}\right)^2 + \left(\frac{M_u}{M_{u0}}\right)^2 = 1 \quad (8)$$

6. APPLICABILITY OF PROPOSED EVALUATION METHOD

Fig.17 and **Fig.18** show comparisons between analytical values and the respective evaluation values for bending moment M_u and local load P_u calculated using Evaluation Equation (8). The filled-in plots on these graphs represent results under pure bending alone or local load alone, respectively. The evaluation results are well accorded with the analytical ones and furthermore, the fact that the evaluation results tend to be lower than the analytical ones

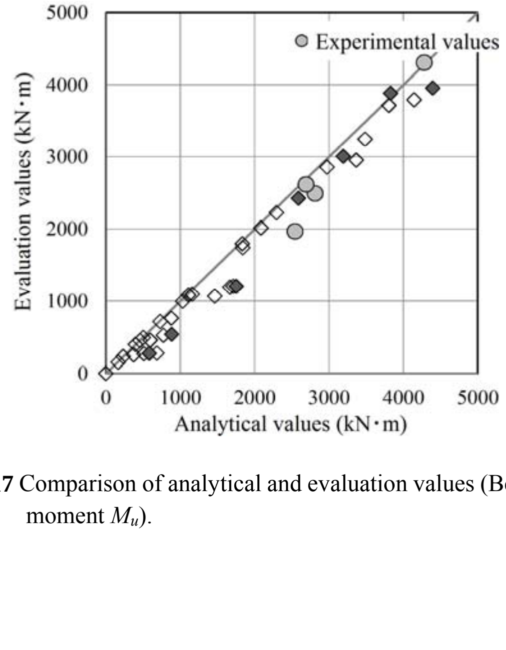


Fig.17 Comparison of analytical and evaluation values (Bending moment M_u).

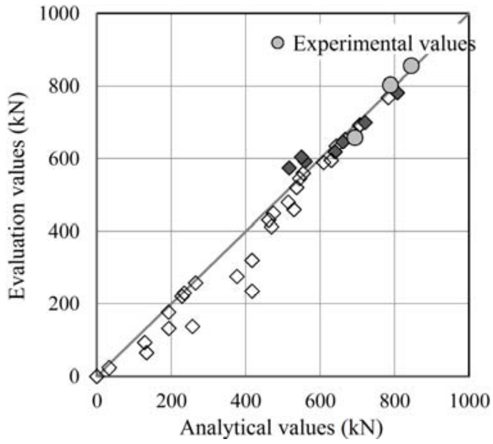


Fig.18 Comparison of analytical and evaluation values (Local load P_u).

shows that the values obtained from the evaluation equation can be estimated on the safe side.

To compare the values obtained from the evaluation equation proposed by this study, circles representing the results of experiments from reference materials[7),10)] are plotted on the graphs. These results also demonstrate that the proposed evaluation method and equation are well accorded with values obtained from experiments.

7. CONCLUSION

As this study aimed to propose a remaining strength evaluation method for plate girders with corrosion under sleepers, the load bearing capacity for bending has been investigated first.

This investigation of bending load bearing capacity demonstrated analytically about the types of collapse triggered by lateral buckling or torsional buckling of compressive flanges. The results of the analysis showed that the bending load bearing capacity of plate girders could be evaluated by taking only compressive flanges from plate girders, that lateral buckling can be obtained from Basler's proposed equation, and that torsional buckling can be obtained from the evaluation equation proposed in this study. The analysis also showed that the bending load bearing capacity obtained from the evaluation equation can depict bending load bearing capacity obtained from past experiments with accuracy and can be estimated on the safe side.

In addition, since bending and local loads act simultaneously on actual railway bridges, elasto-plastic non-linear FEM analysis has been used to investigate the remaining load bearing capacity of plate girders that bear various combined loads. Given these results, this study clarified an interactive curve for plate girders with corrosion under sleepers under combined loads, and proposed a remaining strength evaluation method.

Finally, examining and comparing the values obtained from this evaluation method with the FEM analysis results could verify that this method is an applicable way to calculate the remaining strength of plate girders with corrosion of upper flanges under sleepers.

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